4.9.8 TYPE 68: SHADING OF OPENINGS BY EXTERNAL OBJECTS

General Description

Type68 reads a data file containing the angular heights of obstructions seen from an arbitrary opening and outputs two numbers which describe the time dependent shading of the opening. The first is a 0 or a 1 corresponding to whether beam radiation is blocked or visible. The second is a number between 0 and 1 which gives the fraction of diffuse radiation visible from the opening. A 0 indicates that no diffuse radiation is visible and a 1 indicates that all the diffuse radiation normally visible by the window is indeed visible. The beam radiation calculation is made simply by deciding whether the sun is obstructed by an object at any given time. The diffuse fraction calculation is made by integrating the shading effects seen by the window and dividing by the view from the opening were there no shading objects present. With these dimensionless numbers, the radiation normally incident on the opening can be adjusted to account for shading.

Nomenclature

Underlined letters indicate a vector. Vectors appearing in equations have an arrow over their letter representation.

α	-	Surface angle measured in TRNSYS coordinates (for the northern hemisphere South = 0, East = -90, West = 90, North = ±180)
β	-	the slope of the plane containing an opening
θ	-	The angular height of an obstruction
$\underline{i}$, $\underline{j}$, $\underline{k}$	-	Unit normal vectors in x, y and z respectively
S	-	The plane of an opening
$\underline{n}$S	-	The unit normal vector for S
$\underline{v}$	-	A vector representation of a surface angle
$\underline{p}$	-	The projection of a surface angle vector in the plane S
σ	-	The angle between $\underline{n}$S and $\underline{p}$
h1	-	The height of a spherical zone
h2	-	The height of a spherical cap
S1	-	The surface area of a segment of the spherical zone defined by the plane of the diameter, a height h1 and an angle α
S2	-	The surface area of a segment of the spherical cap defined by a height h2 and an angle α

Mathematical Description

For any given opening (described by a slope and an azimuth) it is necessary to output two numbers. The first is a 0 or a 1 corresponding to whether beam radiation is blocked or visible. The second is a number between 0 and 1 which gives the fraction of diffuse radiation visible from the opening. A 0 indicates that no diffuse radiation is visible and a 1 indicates that all the diffuse radiation normally visible by the window is indeed visible. The fraction is fairly easy to calculate for a vertical or horizontal openings but becomes quite difficult if sloped openings are allowed. A vertical opening has a viewing angle of 180 degrees and a horizontal opening has a view angle of 360 degrees. It is less evident what the viewing angle for an arbitrarily sloped opening is. Consequently, it was decided that all openings would be defined as having a 360 degree view angle and that the plane containing the opening would be considered an obstruction shading the opening.

Obstructions are defined by an angular height as viewed from the opening. Surface angles (α) are defined in an absolute co-ordinate system (as opposed to relative to the opening) and for each one, an angular obstruction height θ is required (Figure 4.9.8.1).

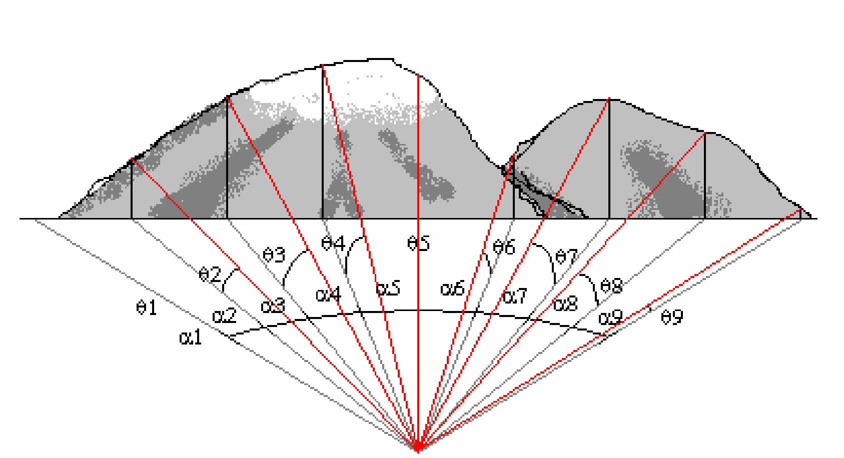

Figure 4.9.8.1: Definition of surface angles (a) and obstruction height angles (q)

For the case of a vertical opening, the plane containing the opening forms θ angles of 90° for all a angles departing to the rear of the window. In the case of a sloped plane, the θ angles departing toward the rear of the opening are not always 90° but follow a the function described by equation 4.9.8.yy. The angle departing directly behind the window forms an angle equal to the slope of the opening (Figure 4.9.8.2)

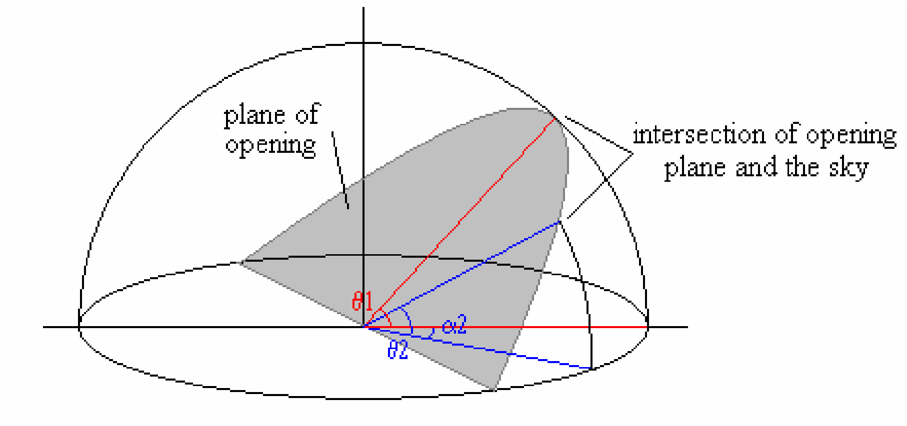

Figure 4.9.8.2: Apparent Obstruction Angles Created by the Plane of a Sloped Opening

Calculation of Visible Diffuse Radiation Fraction

The overall goal of this calculation is to come up with the fraction of diffuse radiation normally visible from the opening that is still visible, given the effects of shading objects. The numerator of the fraction is the amount of sky visible with shading objects. It is found using a call to the DATA subroutine. For each opening identification number and for each direction (α), an angular obstruction height (β) is provided. The format of the data file and an example can be found at the end of the mathematical description.

The denominator of the fraction is the amount of sky normally visible from the opening. A horizontal opening sees the entire sky. A vertical opening sees half the sky, and a tilted opening sees some fraction in between. Instead of trying to calculate how much of the sky is visible, a series of apparent obstruction angles representing the plane of the opening are computed.

For a wall tilted at an angle β, find an expression for the angle between wall and ground for a given surface angle α.

For simplicity sake, we will pretend for now that the plane of the wall faces due south. It can therefore be described by the equation $0X + \tan\beta\, Y + Z = 1$. We will call the plane, as shown in figure 4.9.8.3, S.

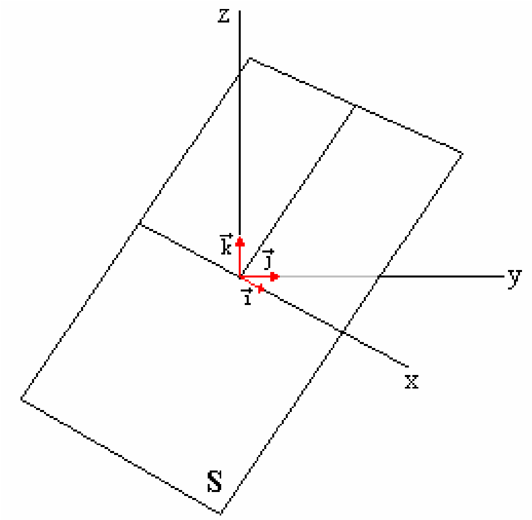

Figure 4.9.8.3: Coordinate System for the Plane Containing an Opening

The normal vector for plane S has the direction nS{0,tanβ,1}
The length of the normal vector is given by the determinant of the vector nS as

$$|\vec{n}S| = \sqrt{0^2 + (\tan b)^2 + 1^2} = \sqrt{1 + (\tan b)^2} \tag{4.9.8.1}$$

The unit normal vector to S is thus

$$\vec{n}S = \frac{1}{\sqrt{1 + (\tan b)^2}}\left(\tan b\vec{j} + \vec{k}\right) \tag{4.9.8.2}$$

A vector v representing each surface angle α is projected onto plane S by the following formula in which p is the resulting vector in S and in which k is a constant.

$$\vec{p} = \vec{v} - k\vec{n}S \tag{4.9.8.3}$$

To find k, we dot both sides of the equation with nS. Since p and nS are perpendicular, their dot product is 0. nS dotted with itself is unity. Consequently, the formula for k is

$$k = \vec{v} \bullet \vec{n}S \tag{4.9.8.4}$$

The vector v exists in the plane of the horizontal so it only has x and y components. The formula for v comes directly from the definition of the tangent in a right triangle

$$\vec{v} = \left(\tan a\vec{i} + \vec{j}\right) \tag{4.9.8.5}$$

solving for k gives

$$k = (\tan a)(0) + (1)\left(\frac{\tan b}{\sqrt{1+(\tan b)^2}}\right) + (0)(1) = \frac{\tan b}{\sqrt{1+(\tan b)^2}} \qquad (4.9.8.6)$$

Now we can write and simplify a vector for p

$$\vec{p} = (\tan a\vec{i} + \vec{j}) - \left(\frac{\tan b}{\sqrt{1+(\tan b)^2}}\right)\left(\frac{1}{\sqrt{1+(\tan b)^2}}\right)(\tan b\vec{j} + \vec{k}) \qquad (4.9.8.7)$$

$$\vec{p} = (\tan a\vec{i} + \vec{j}) - \left(\frac{\tan b}{1+(\tan b)^2}\right)(\tan b\vec{j} + \vec{k}) \qquad (4.9.8.8)$$

$$\vec{p} = \tan a\vec{i} + \left(1 - \frac{(\tan b)^2}{1+(\tan b)^2}\right)\vec{j} + \frac{\tan b}{1+(\tan b)^2}\vec{k} \qquad (4.9.8.9)$$

The next step is to find the angle that the projection vector p makes with the plane of the horizontal. The angle is found by taking the dot product of p and the unit normal of the horizontal plane.

The unit normal vector to the horizontal is {0,0,1}

$$\vec{n}H \bullet \vec{p} = |\vec{n}H||\vec{p}|\cos s \qquad (4.9.8.10)$$

expanding gives

$$\{0,0,1\} \bullet \left\{\tan a, 1 - \frac{(\tan b)^2}{1+(\tan b)^2}, \frac{\tan b}{1+(\tan b)^2}\right\} = \sqrt{0^2+0^2+1^2}\sqrt{(\tan a)^2 + \left(1 - \frac{(\tan b)^2}{1+(\tan b)^2}\right)^2 + \left(\frac{\tan b}{1+(\tan b)^2}\right)^2}\cos s \qquad (4.9.8.11)$$

which can be simplified to

$$\frac{\tan b}{1+(\tan b)^2} = \sqrt{(\tan a)^2 + \left(1 - \frac{(\tan b)^2}{1+(\tan b)^2}\right)^2 + \left(\frac{\tan b}{1+(\tan b)^2}\right)^2}\cos s \qquad (4.9.8.12)$$

Equation 4.9.xx.12 can be solved for the angle between the projection of a surface angle α in S and a vertical line (the normal to the horizontal plane). The angle σ is subtracted from 90 to arrive at the angle that the projection makes with the horizontal plane. The result is called θ. At this stage, we have a set of apparent obstruction height angles for the plane of the opening. This set is compared with the data provided in the file for each surface angle α.

There is of course one more complication. Simply comparing two obstruction heights and dividing them to find the fraction of sky that remains visible for that surface angle does not take into account the fact that each surface angle in reality represents a wedge of sky; an obstruction angle height of θ = 45° does not mean that for the given α, half the sky is visible. Instead, we need to compare the two surface areas S1 and S2 as shown in figure 4.9.8.4.

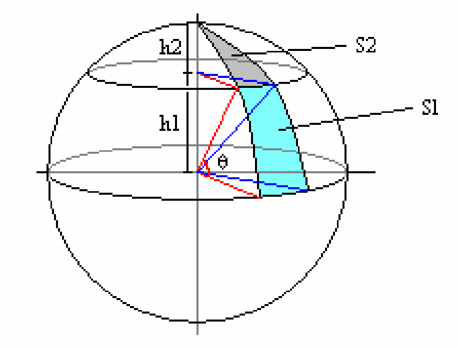

Figure 4.9.8.4: Segment Areas

The fraction of sky visible for a given a is

$$f = \frac{S2}{S2 + S1} \tag{4.9.8.13}$$

Because we are working with a sphere, we can write that

$$h1 + h2 = r \tag{4.9.8.14}$$

Furthermore, a right triangle is formed by the vertical axis of the sphere, a radius and the plane containing the base of the segment S2. Consequently, it can be written that

$$h1 = r\cos(2p - q) \tag{4.9.8.15}$$

in which θ is in radians.

The areas S1 (a segment of a spherical zone) and S2 (a segment of a spherical cap) both have the same basic formula. γ is measured in radians.

$$S1 = grh1$$
$$S2 = grh2 \qquad (4.9.8.16)$$

Using the results of equations 4.9.8.14, 4.9.8.15 and 4.9.8.16, equation 4.9.8.13 can be rewritten as

$$f = 1 - \cos(2p - q) \qquad (4.9.8.17)$$

θ is again in radians

Now that all the math is in place, two θ sums over the set of surface angles (α) are made and divided. The numerator contains the set θ angles from the data file as applied to equation 4.9.8.17. The denominator contains the set of θ angles found using equation 4.9.8.12 as applied to equation 4.9.8.17.

$$f = \frac{\sum_{a=-p}^{p} 1 - \cos(2p - q_{shaded})_a}{\sum_{a=-p}^{p} 1 - \cos(2p - q_{unshaded})_a} \qquad (4.9.8.18)$$

Calculation of Visible Beam Radiation

To detect whether the beam radiation is behind an obstruction, Type68 takes the sun’s position (solar azimuth angle and solar altitude angle), and compares them to the information in the data file. If, for a given a, the solar altitude angle is less than the corresponding obstruction height angle, beam radiation is said to be blocked.

The Data File

Type68 centers around a data file that contains the angular heights (θ) of obstructions seen from an opening. The type makes use of a call to the DATA subroutine and therefore there are certain restrictions placed on the Type. Currently, the DATA subroutine is only able to read 20 values of α and 10 values of θ. These limitations keep the size of the TRNSYS dll manageable. They can of course be increased to suit the user’s needs.

The file format should be as follows.

OPEN_ID1 OPEN_ID2 … OPEN_Idn
SLP1 SLP2 … SLPn
AZ1 AZ2 … AZn
-180 -162 -144 -126 -108 -90 -72 -54 -36 -18 0 18 36 54 72 90 108 126 144 162

θ for OPEN_ID1 at α = -180
θ for OPEN_ID1 at α = -162
…
θ for OPEN_ID1 at α = 162
θ for OPEN_ID2 at α = -180
θ for OPEN_ID2 at α = -162
…
θ for OPEN_ID2 at α = 162
…
θ for OPEN_IDn at α = -180
θ for OPEN_IDn at α = -162
…
θ for OPEN_IDn at α = 162

Opening ID numbers should be in increasing order.

TRNSYS Component Configuration:

PARAMETER NO.			DESCRIPTION
1	LU	-	Logical unit number of file containing data
2	N_{opens}	-	Number of openings in the file
3	$N_{surfangs}$	-	Number of surface angles for which obstruction height angles are provided

INPUT NUMBER			DESCRIPTION
1	γ	-	Solar azimuth angle (degrees)
2	θ_z	-	Solar zenith angle (degrees)
3	Ib_1	-	Beam radiation on surface 1 (kJ/hr-m^2)
4	Id_1	-	Diffuse radiation on surface 1 (kJ/hr-m^2)
…	…		…
2*PAR(2)+1	Ib_n	-	Beam radiation on surface n (kJ/hr-m^2)
2*PAR(2)+2	Id_n	-	Diffuse radiation on surface n (kJ/hr-m^2)

OUTPUT NUMBER			DESCRIPTION
1	α_s	-	Solar altitude angle (degrees)
2	$f_{beam,1}$	-	Flag=1 if beam radiation is visible from opening 1 (-)
3	Ibs_1	-	Shaded beam radiation for surface 1 (kJ/hr-m^2)
4	$f_{diffuse,1}$	-	Fraction of diffuse radiation that is visible for opening 1 (-)

5	Ids_1	-	Shaded diffuse radiation for surface 1 (kJ/hr-m^2)
6	Its_1	-	Shaded total radiation for surface 1 (kJ/hr-m^2)
7	$f_{beam,2}$	-	Flag=1 if beam radiation is visible from opening 2 (-)
8	Ibs_2	-	Shaded beam radiation for surface 2 (kJ/hr-m^2)
9	$f_{diffuse,2}$	-	Fraction of diffuse radiation that is visible for opening 2 (-)
10	Ids_2	-	Shaded diffuse radiation for surface 2 (kJ/hr-m^2)
11	Its_2	-	Shaded total radiation for surface 2 (kJ/hr-m^2)
…	…		…
5*(PAR(2)-1)+2	$f_{beam,n}$	-	Flag=1 if beam radiation is visible from opening n (-)
5*(PAR(2)-1)+3	Ibs_n	-	Shaded beam radiation for surface n (kJ/hr-m^2)
5*(PAR(2)-1)+4	$f_{diffuse,n}$	-	Fraction of diffuse radiation that is visible for opening n (-)
5*(PAR(2)-1)+5	Ids_n	-	Shaded diffuse radiation for surface n (kJ/hr-m^2)
5*(PAR(2)-1)+6	Its_n	-	Shaded total radiation for surface n (kJ/hr-m^2)